PHOTOGRAPH THIS SHEET

AD A080823

DTIC ACCESSION NUMBER

LEVEL

INVENTORY

FTD-ID(RS)T-0371-79

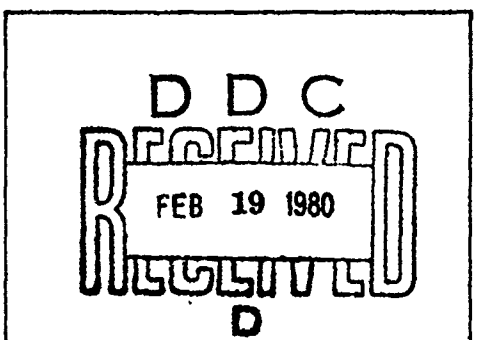
DOCUMENT IDENTIFICATION

DISTRIBUTION STATEMENT A
Approved for public release;
Distribution Unlimited

DISTRIBUTION STATEMENT

ACCESSION FOR	
NTIS	GRA&I
DTIC	TAB
UNANNOUNCED	
JUSTIFICATION	
BY	
DISTRIBUTION /	
AVAILABILITY CODES	
DIST	AVAIL AND/OR SPECIAL
A	

DISTRIBUTION STAMP



DATE ACCESSIONED

DATE RECEIVED IN DTIC

PHOTOGRAPH THIS SHEET AND RETURN TO DTIC-DDA-2

DTIC FORM OCT 79 70A

DOCUMENT PROCESSING SHEET

FTD-ID(RS)T-0371-79

FOREIGN TECHNOLOGY DIVISION

RELAXATION OF MOMENTUM AND ENERGY OF AN ELECTRON IN A CRYSTAL IV. INELASTIC SCATTERING ON OPTICAL PHONONS

By

P. A. Kazlauskas, I. B. Levinson

EDITED TRANSLATION

FTD-ID(RS)T-0371-79 21 March 1979

MICROFICHE NR: 24D-79-C-000417

RELAXATION OF MOMENTUM AND ENERGY OF AN ELECTRON IN A CRYSTAL IV. INELASTIC SCATTERING ON OPTICAL PHONONS

By: P. A. Kazlauskas, I. B. Levinson

English pages: 10

Source: Litovskiy Fizicheskiy Sbornik, Vol. 6, Nr. 4, 1966, pp. 513-522

Country of Origin: USSR
Translated by: Robert D. Hill
Requester: AFWL/IN

PREPARED BY:

TRANSLATION DIVISION
FOREIGN TECHNOLOGY DIVISION
WP-AFB, OHIO.

U. S. BOARD ON GEOGRAPHIC NAMES TRANSLITERATION SYSTEM

Block	Italic	Transliteration	Block	Italic	Transliteration
А а	*А а*	A, a	Р р	*Р р*	R, r
Б б	*Б б*	B, b	С с	*С с*	S, s
В в	*В в*	V, v	Т т	*Т т*	T, t
Г г	*Г г*	G, g	У у	*У у*	U, u
Д д	*Д д*	D, d	Ф ф	*Ф ф*	F, f
Е е	*Е е*	Ye, ye; E, e*	Х х	*Х х*	Kh, kh
Ж ж	*Ж ж*	Zh, zh	Ц ц	*Ц ц*	Ts, ts
З з	*З з*	Z, z	Ч ч	*Ч ч*	Ch, ch
И и	*И и*	I, i	Ш ш	*Ш ш*	Sh, sh
Й й	*Й й*	Y, y	Щ щ	*Щ щ*	Shch, shch
К к	*К к*	K, k	Ъ ъ	*Ъ ъ*	"
Л л	*Л л*	L, l	Ы ы	*Ы ы*	Y, y
М м	*М м*	M, m	Ь ь	*Ь ь*	'
Н н	*Н н*	N, n	Э э	*Э э*	E, e
О о	*О о*	O, o	Ю ю	*Ю ю*	Yu, yu
П п	*П п*	P, p	Я я	*Я я*	Ya, ya

*ye initially, after vowels, and after ъ, ь; e elsewhere.
When written as ё in Russian, transliterate as yё or ё.

RUSSIAN AND ENGLISH TRIGONOMETRIC FUNCTIONS

Russian	English	Russian	English	Russian	English
sin	sin	sh	sinh	arc sh	$\sinh^{-1}$
cos	cos	ch	cosh	arc ch	$\cosh^{-1}$
tg	tan	th	tanh	arc th	$\tanh^{-1}$
ctg	cot	cth	coth	arc cth	$\coth^{-1}$
sec	sec	sch	sech	arc sch	sech^{-1}
cosec	csc	csch	csch	arc csch	csch^{-1}

Russian	English
rot	curl
lg	log

RELAXATION OF MOMENTUM AND ENERGY OF AN ELECTRON IN A CRYSTAL

IV. INELASTIC SCATTERING ON OPTICAL PHONONS

P.A. Kazlauskas, I.B. Levinson
(Submitted 16 March 1966)

Computed are the lifetimes, times of the relaxation and fluctuation of the momentum and velocity, and characteristics of the relaxation and fluctuation of the energy with deformation and polarization scattering on optical phonons without dispersion and an arbitrary isotropic zonal structure.

Introduction

The inelastic scattering on optical phonons of electrons with an energy of $\varepsilon \sim \hbar\omega_0$ is a very common scattering mechanism. It is known that in this case it is impossible to reduce the term from the collision in the kinetic equation to a differential form, and it is also impossible to introduce the relaxation time into the kinetic equation. However, we can determine the times of relaxation (and fluctuation) for a test electron, which are computed in this work. These times give a representation about the rate of relaxation of the energy and momentum; furthermore, they enter into the balance equation, from which the electron temperature and drift momentum of the shifted Maxwellian distribution are defined.

All the designations and concepts unspecified in the work follow the works [1], [2] and [3], which are cited as I, II and III.

1. Formulae for the characteristic times of the test electron

In addition to the times of relaxation and fluctuation of momentum examined in § 3 [1], let us introduce similar times for the velocity:

time of relaxation of velocity

$$\frac{v(\varepsilon)}{\tau'(\varepsilon)} = \int (dp')\ W(p, p')\,[v(p)-v(p')]_u, \qquad (1.1)$$

time of deviation of velocity

$$\frac{v^2(\varepsilon)}{\tau'_\perp(\varepsilon)} = \int (dp')\ W(p, p')\,[v(p)-v(p')]^2_\perp, \qquad (1.2)$$

time of longitudinal fluctuation of velocity

$$\frac{v^2(\varepsilon)}{\tau'_{\parallel}(\varepsilon)} = \int (dp')\ W(p, p')\,[v(p)-v(p')]^2_{\parallel}. \qquad (1.3)$$

If by analogy with (3.5) of I, we introduce the times τ'_{in} according to

$$\frac{v^n(\varepsilon)}{\tau'_{in}(\varepsilon)} \int_0^\infty d\varepsilon'\ g(\varepsilon')\ W_i(\varepsilon, \varepsilon')\ v^n(\varepsilon'), \qquad (1.4)$$

the times (1.1)-(1.3) are expressed by τ'_{in} according to the same precise formulae as (3.7)-(3.9) of I.

The times of the relaxation and fluctuation of the momentum and velocity coincide in two cases: for the parabolic band with an arbitrary nature of scattering and for an almost elastic scattering with an arbitrary zonal structure. In the first case, the coninciden̈ce is connected with the "uniformity" of the parabolic band and, in the second case, with the fact that with an almost elastic scattering, a small part of the band, which can always be considered "uniform," is important. Understood by uniformity here is the independence of quantity $p(\varepsilon)/v(\varepsilon)$ of ε.

The times of relaxation and fluctuation of the momentum and velocity on optical phonons are easily calculated if we disregard the dispersion of the latter. The results are given in Table (1.1) where denoted is

$$\Lambda^{\pm}(\varepsilon) = \ln\left|\frac{p(\varepsilon)+p(\varepsilon\mp\hbar\omega_0)}{p(\varepsilon)-p(\varepsilon\mp\hbar\omega_0)}\right|. \qquad (1.5)$$

Under the assumption made, the electron is replaced or loses its energy by constant portions $\hbar\omega_0$. in view of which

$$Q^{\pm}(\varepsilon) = \hbar\omega_0 \frac{1}{\tau_0^{\pm}(\varepsilon)}. \tag{1.6}$$

$$D^{\pm}(\varepsilon) = \frac{1}{2}(\hbar\omega_0)^2 \frac{1}{\tau_0^{\pm}(\varepsilon)}. \tag{1.7}$$

Results for the parabolic band are listed in Table 1.2, where (1.5) takes the form

$$A^{+}(\varepsilon) = 2\,\mathrm{Arch}\left(\frac{\varepsilon}{\hbar\omega_0}\right)^{\frac{1}{2}}. \tag{1.8}$$

$$A^{-}(\varepsilon) = 2\,\mathrm{Arsh}\left(\frac{\varepsilon}{\hbar\omega_0}\right)^{\frac{1}{2}}. \tag{1.9}$$

Some of the results for the parabolic band, sometimes in a different form, were obtained earlier by different authors: τ and Q for PO in [6] and for DO in [7].

Let us note that for electrons with an energy somewhat greater than the energy of the optical phonon, i.e., $\varepsilon - \hbar\omega_0 \ll \hbar\omega_0$. the dependence of the emission times on ε is identical for the PO and DO mechanisms. The fact is that with the emission of the phonon, such an electron is almost stopped, and therefore all the phonons being emitted have an identical momentum $q \approx p(\varepsilon)$. and the dependence of B(q) on q does not appear.

Let us compare the characteristic quantities given in Table 1.1 and Table 1.2 and also in (1.6) and (1.7) with results of calculations by other methods. For the elastic scattering, when $\varepsilon \gg \hbar\omega_0$. they, obviously, are converted into quantities which enter into the kinetic equation and given in tables 3.2 II and 3.4 II, respectively. For electrons with $\varepsilon < \hbar\omega_0$. when only absorption at low temperatures $kT \ll \hbar\omega_0$ is possible, it makes sense to compare the lifetimes and times of relaxation of the momentum with results of the calculation according to the composite scattering conducted in III, when it is considered that the instantaneous remission follows behind the absorption. As follows from the general theory, the lif times τ_0 coincide; however, the times of relaxation of the momentum τ are different, although coinciding in order of magnitude. This difference is associated wtih the

fact that the instantaneous remission changes the direction of the finite momentum, thereby changing the increment of the momentum in the act of the composite scattering in comparison with the simple absorption. For the mechanism DO, where the scattering is symmetric, this difference disappears.

It makes no sense to compare Q and D with the corresponding quantities for the composite scattering, since in the latter case the dispersion of the optical phonons is important.

Certain "relaxation times" for the inelastic PO scattering were calculated by means of the Kubo method in [4] for the parabolic band and in [5] for the keynov band; here the virtual transfers were disregarded. Result [4] coincides with the time τ from Table 1.2. Thus the calculation according to the Kubo method has a simple physical meaning - this is simply the time of the relaxation of the test particle. However, it is interesting to note that the result [5] for the nonparabolic zone coincides with the relaxation time of the velocity τ' from Table 1.1 and not with the relaxation time of the momentum τ.

2. Relaxations of momentum and energy as a function of the energy of the test electron

The general character of the dependence on ε of the most important quantities $1/\tau(\varepsilon)$ and $Q(\varepsilon)$ for the paraboloic band is shown on Figure 2.1 (high temperatures $kT \gg \hbar\omega_0$) and on Fig. 2.2 (low temperatures $kT \ll \hbar\omega_0$).

All the curves are characterized by an increase when $\varepsilon \to \infty$ for DO and a decrease when $\varepsilon \to \infty$ for PO. This is obviously connected with the short-acting nature of DO of the mechanism and long-range interacting nature of PO of the mechanism. When $\varepsilon = \hbar\omega_0$ all the curves have a sharp bend connected with the inclusion of emission scattering. At low temperatures this bend is accompanied by a sharp increase in $\exp\{\hbar\omega_0/kT\}$ times, owing to the effective spontaneous emission.

Let us examine first the detailed dependence $1/\tau(\varepsilon)$. For DO it is monotonic, and for PO it has a maximum in the region of $\varepsilon \sim \hbar\omega_0$ both for the high and for the low temperatures. For the

high temperatures, with the transfer from region $\varepsilon \sim \hbar\omega_0$ to region $\varepsilon \sim kT$ the quantity $1/\tau(\varepsilon)$ in order increases (for DO) or decreases (for PO) $(\hbar\omega_0/kT)^{\frac{1}{2}}$ times.

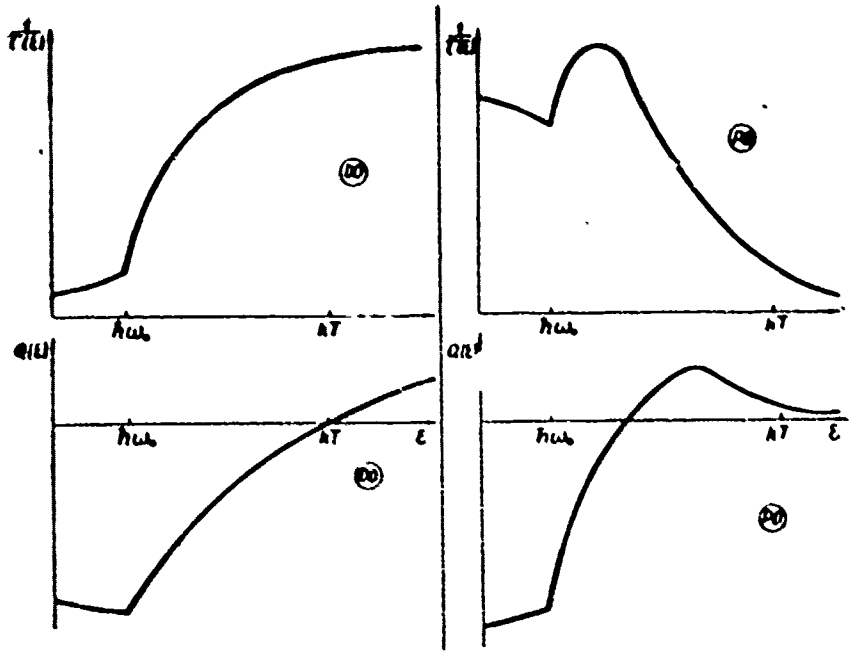


Fig. 2.1. Dependence of $1/\tau(\varepsilon)$ and $Q(\varepsilon)$ on ε at high temperatures.

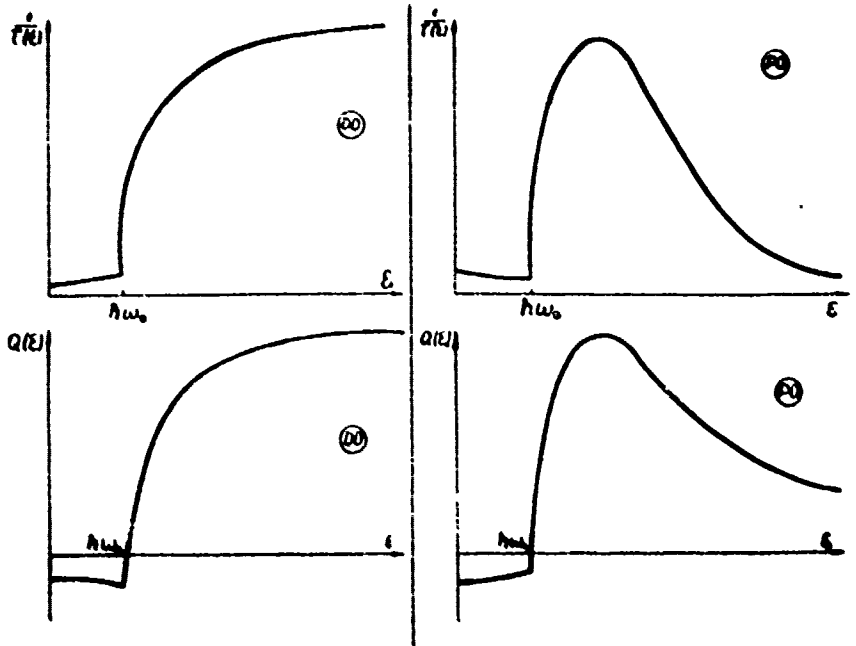


Fig. 2.2. Dependence of $1/\tau(\varepsilon)$ and $Q(\varepsilon)$ on ε at low temperatures.

Table 1.1 Characteristic times of inelastic scattering on optical phonons without dispersion

Table 1.2 Times of inelastic scattering on optical phonons without dispersion (parabolic band)

The dependence $Q(\varepsilon)$ is more complex. This quantity changes sign and, furthermore, has a maximum for PO.

When $\varepsilon<\hbar\omega_0$ there occurs only the absorption, and therefore $Q(\varepsilon)=Q^-(\varepsilon)<0$. With the transfer through $\varepsilon\sim\hbar\omega_0$ there appears an emission component $Q^+(\varepsilon)>0$, and at low temperatures Q^- contains in comparison with Q^+ a small factor $\exp\{-\hbar\omega_0/kT\}$, and, therefore, the appearance of the large component Q^+ leads to a rapid change in the sign of Q, i.e., $\varepsilon^*=\hbar\omega_0$. Actually, $Q^+(\varepsilon)$ when $\varepsilon\to\hbar\omega_0$ is small as $(\varepsilon-\hbar\omega_0)^{\frac{1}{2}}$, and, therefore, ε^* proves to be somewhat larger than $\hbar\omega_0$; their difference is of the order of $\hbar\omega_0\exp\{-\hbar\omega_0/kT\}$. At high temperatures Q^+ and Q^- are of the same order; therefore, the appearance of the emission does not lead to a rapid change in the sign of Q, and it is found that $\varepsilon^*>\hbar\omega_0$. Speaking differently, a change in the sign of Q occurs in the region of the elastic scattering, and it is more convenient to examine it as a result of the competition of the spontaneous and induced losses in Q^T and Q^0 (§1 of III). For mechanisms of scattering without distinctive features this gives $\varepsilon^*\sim kT$, which, in particular, for DO is $\varepsilon^*=kT$. For the long-range interacting mechanism of PO, we get $\varepsilon^*\sim kT/\Lambda<kT$, where $\Lambda\gg1$ is the value of $\Lambda(\varepsilon)$ when $\varepsilon=kT$. The reason for this can be explained in the following way. The emission and absorption of the phonons with small q does not contribute to Q^T, since these processes are compensated (see Fig. 3, I); meanwhile, in Q^0 the emission of the phonons with small q gives a contribution. Therefore, Q^0, unlike Q^T, proves to be abnormally large, and Q^0 is compared with Q^T at a value of ε less than kT.

The maximum of $Q(\varepsilon)$ for PO takes places in the same region of energies as does the change in the sign, i.e., when $\varepsilon\sim\hbar\omega_0$ for low temperatures and $\varepsilon\sim kT/\Lambda$ for high temperatures. For high temperatures with the transfer from $\varepsilon\sim\hbar\omega_0$ to $\varepsilon\sim kT$ the quantity $Q(\varepsilon)$ is decreased in order by $(\hbar\omega_0/kT)^{\frac{1}{2}}$ times for DO and by $(\hbar\omega_0/kT)^{\frac{1}{2}}\Lambda$ times for PO. The height of the maximum for the PO is $\Lambda^{\frac{1}{2}}$ larger than the value when $\varepsilon\sim kT$.

3. Efficiency of the relaxation and fluctuation of momentum, velocity and energy

The efficiency of the relaxation and fluctuation of momentum is described by the quantities

$$\alpha = \frac{\tau_0}{\tau}, \quad \alpha_{\parallel} = \frac{\tau_0}{\tau_{\parallel}}, \quad \alpha_{\perp} = \frac{\tau_0}{\tau_{\perp}}. \tag{3.1}$$

which show what portion of the momentum relaxes or fluctuates with one event of scattering. Similar quantities can be introduced also for the velocity

$$\alpha' = \frac{\tau_0}{\tau'}, \quad \alpha'_{\parallel} = \frac{\tau_0}{\tau'_{\parallel}}, \quad \alpha'_{\perp} = \frac{\tau_0}{\tau'_{\perp}}. \tag{3.2}$$

From Table 1.1 and Table 1.2 it is seen that in the region of the elastic scattering, when $\varepsilon \gg \hbar\omega_0$ we have for the PO-mechanism

$$\alpha(\varepsilon) = \alpha'(\varepsilon) \sim \alpha_{\parallel}(\varepsilon) = \alpha'_{\parallel}(\varepsilon) \sim \alpha_{\perp}(\varepsilon) = \alpha'_{\perp}(\varepsilon) \sim \Lambda^{-1}(\varepsilon) \ll 1. \tag{3.3}$$

This is connected with the long-range interacting nature of the PO-mechanism and indicates that with elastic scattering it dominates the interaction with the long-wave phonons; the latter leads to the inefficiency of the relaxation and fluctuation of the momentum and velocity. For the short-range interacting DO mechanism, where the interaction with the long-wave phonons is weak, this effect is absent, and all α in (3.3) is of the order of unity.

In the region of the inelastic scattering, when $\varepsilon \sim \hbar\omega_0$ both for the PO and for the DO mechanism

$$\alpha(\varepsilon) \sim \alpha'(\varepsilon) \sim \alpha_{\parallel}(\varepsilon) \sim \alpha'_{\parallel}(\varepsilon) \sim \alpha_{\perp}(\varepsilon) \sim \alpha'_{\perp}(\varepsilon) \sim 1, \tag{3.4}$$

since with such energy the laws of the conservation of momentum and energy forbid the emission and absorption of the phonons with small q (Fig. 3, I).

In the region of highly inelastic scattering, when $\varepsilon \ll \hbar\omega_0$ both for the PO and for the DO mechanism

$$\alpha(\varepsilon) \sim \alpha'(\varepsilon) \sim 1,$$

$$\alpha_{\parallel}(\varepsilon) \sim \alpha'_{\parallel}(\varepsilon) \sim \alpha_{\perp}(\varepsilon) \sim \alpha'_{\perp}(\varepsilon) \sim \frac{\hbar\omega_0}{\varepsilon} \gg 1. \tag{3.5}$$

This can be explained in the following way. In absorbing the phonon, the electron increases its energy up to $\hbar\omega_0$ as a result of which the absolute magnitude of its finite momentum proves to be much more than the initial momentum. It is precisely,

therefore, that the fluctuation of the momentum is abnormally great. This has no effect on the relaxation, since the great deviations in reverse are compensated by the great deviations forward. This compensation is precise, since when $\varepsilon \to 0$ the momentum q, as is seen from (2.5) I, stops being dependent upon the angle of scattering χ, and the scattering is made symmetric at any dependence of B(q) on q.

The situation connected with the long-range interacting nature of PO scattering in the region of energy $\varepsilon \gg \hbar\omega_0$, is similar to the situation with scattering on Coulomb centers; however, there is a significant difference. With Coulomb scattering, by introducing the small parameter of cutoff $\xi \to 0$ (for example, due to the Debye shielding), we have that τ_0 is of the order of ξ, τ, $\tau_\parallel$, $\tau_\perp$ is of one order of $|\ln \xi|^{-1}$, and the times of the higher order (connected with a change in the momentum $|\mathbf{p}-\mathbf{p}'|^3$, $|\mathbf{p}-\mathbf{p}'|^4$...) are finite. With PO scattering only τ_0 of the order of Λ^{-1} proves to be small, and τ, $\tau_\parallel$ and $\tau_\perp$ have the same order as the times of higher orders. The reason for this is the more rapid decrease in the dipole field of PO scattering in comparison with the Coulomb field. For this reason, the collision term of the kinetic equation for the PO scattering can not be reduced to the Fokker-Planck form in all three components of the momentum.

Efficiencies of the relaxation and fluctuation of energy are determined by the relations

$$\delta = \frac{\tau_0}{\tau}, \quad \bar{\delta} = \frac{\tau_0}{\bar{\tau}}. \tag{3.6}$$

Using (1.7), it is possible to show that

$$\bar{\delta}(\varepsilon) = \frac{1}{2}\left(\frac{\hbar\omega_0}{\varepsilon}\right)^2. \tag{3.7}$$

Obviously, this is simply a square of the relative change in energy in any event of scattering, irrespective of whether it is emission or absorption. For the efficiency of the relaxation of energy in the region of elastic scattering, where $\varepsilon \gg \hbar\omega_0$, we get

$$\bar{\delta}(\varepsilon) \sim \frac{1}{2N_0+1}\,\frac{\hbar\omega_0}{\varepsilon_1}. \tag{3.8}$$

The second factor is the relative change in energy in the event

c scattering; the first factor is connected with the fact that from $2N_0+1$ events of scattering, N_0 absorption and N_0+1 emission, on the average only one excessive event of emission leads to the energy relaxation. In the region of the highly inelastic scattering $\varepsilon \ll \hbar\omega_0$ we get

$$\delta(\varepsilon) \sim 1, \quad DO, \quad PO, \quad \text{low temperatures;}$$

$$\left.\begin{aligned} \delta(\varepsilon) &\sim \frac{\hbar\omega_0}{kT}, \quad DO \\ \delta(\varepsilon) &\sim \frac{\hbar\omega_0}{kT}\Lambda, \quad PO \end{aligned}\right\} \text{high temperatures.}$$

The meaning of these results can be understood if we keep in mind that $\delta(\varepsilon)$ determines the efficiency of the relaxation of energy ε to energy ε^*. At low temperatures $\varepsilon^* \sim \hbar\omega_0$, and the electron with $\varepsilon \ll \hbar\omega_0$ reaches $\varepsilon \sim \varepsilon^*$ with one event of absorption. At high temperatures the number of events necessary for this is of the order of $\varepsilon^*/\hbar\omega_0$, which concurs with (3.9).

Institute of Physics and Mathematics of the Academy of Sciences of the Lithuanian SSR

Bibliography

1. П. А. Каллаускас, И. Б. Лесинсон. Liet. fiz. rinkinys, 6, 33 (1966) (цитируется как I)
2. П. А. Каллаускас, И. Б. Лесинсон. Liet. fiz. rinkinys, 6, 233 (1966) (цитируется как II)
3. П. А. Каллаускас, И. Б. Лесинсон, Р. Э. Мажуолите. Liet. fiz rinkinys, 6, 377 (1966) (цитируется как III)
4. А. Г. Самойлович, М. И. Каминер и Л. Л. Коренблит. ФТТ, сборник статей II (1959)
5. Г. И. Гусева и И. М. Цидильковский. ФТТ, 4, 9(1962)
6. R. Stratton. Proc. Phys. Soc., A246, 406 (1958)
7. E. M. Conwell and A. L. Brown. J. Phys. Chem. Solids, 18 pp. 208--217 (1960)

Summary

Velocity relaxation and fluctuation times τ', $\tau'_{||}$, $\tau'_{\perp}$ are given by (1.1) — (1.3). When the scattering is elastic, or the band is parabolic, these velocity characteristic times coincide with corresponding momentum characteristic times τ, $\tau_{||}$, $\tau_{\perp}$ velocity and momentum characteristic time, energy loss power Q and energy fluctuation "power" D are given in Table 1.1 (nonparabolic band) and Table 1.2 (parabolic band).

For the most important quantities — momentum relaxation time τ and energy loss power Q the dependence on electron energy ε is illustrated in Fig. 2.1 (high lattice temperatures $kT > \hbar\omega_0$) and Fig 2.2 (low temperatures $kT < \hbar\omega_0$).

DISTRIBUTION LIST

DISTRIBUTION DIRECT TO RECIPIENT

ORGANIZATION		MICROFICHE
A205	DMATC	1
A210	DMAAC	2
B344	DIA/RDS-3C	9
C043	USAMIIA	1
C509	BALLISTIC RES LABS	1
C510	AIR MOBILITY R&D LAB/FIO	1
C513	PICATINNY ARSENAL	1
C535	AVIATION SYS COMD	1
C591	FSTC	5
C619	MIA REDSTONE	1
D008	NISC	1
H300	USAICE (USAREUR)	1
P005	DOE	1
P050	CIA/CRB/ADD/SD	1
NAVORDSTA (50L)		1
NASA/KSI		1
AFIT/LD		1
LLL/Code L-389		1
NSA/1213/TDL		2

ORGANIZATION		MICROFICHE
E053	AF/INAKA	1
E017	AF/RDXTR-W	1
E403	AFSC/INA	1
E404	AEDC	1
E408	AFWL	1
E410	ADTC	1
	FTD	
	CCN	1
	ASD/FTD/NIIS	3
	NIA/PHS	1
	NIIS	2